1. Evaluate each of the following integrals.

(i) $\int e^{2x-5}\,dx$ [3]

$\frac{1}{2}e^{2x-5} + c$

(ii) $\int \frac{1}{2x^2+5}\,dx$ [7]

$\int \frac{1}{5\left(1+\frac{2}{5}x^2\right)}\,dx = \frac{1}{5}\int \frac{1}{1+\left(\sqrt{\frac{2}{5}}x\right)^2}\,dx$

$= \frac{1}{5}\cdot\frac{1}{\sqrt{\frac{2}{5}}}\tan^{-1}\left(\sqrt{\frac{2}{5}}x\right) + c$

$= \frac{\sqrt{10}}{10}\tan^{-1}\left(\frac{\sqrt{10}}{5}x\right) + c.$

(iii) $\int_0^{\frac{1}{6}\pi} (\cos x + 2\sin x)^2\,dx$ [7]

$\int_0^{\frac{\pi}{6}} \left(\cos^2 x + 4\sin x\cos x + 4\sin^2 x\right) dx$

$= \int_0^{\frac{\pi}{6}} \left[1 + 3\left(\frac{1-\cos 2x}{2}\right) + 2\sin 2x\right] dx$

$= \left[\frac{5}{2}x - \frac{3}{2}\cdot\frac{1}{2}\sin 2x + 2\left(-\frac{1}{2}\right)\cos 2x\right]_0^{\frac{\pi}{6}}$

$= \left[\frac{5}{12}\pi - \frac{3}{4}\times\frac{\sqrt{3}}{2} - \frac{2}{2}\cdot\frac{1}{2}\right] - \left[0 - 0 - \cancel{1}\right] = \overset{1}{\cancel{2}} + \frac{5}{12}\pi - \frac{3\sqrt{3}}{8} - \cancel{4}\,\frac{1}{2}$

$= \underset{\frac{1}{2}}{\cancel{4}} + \frac{5}{12}\pi - \frac{3}{8}\sqrt{3}.$

$= 1.16$

2. Let R be the region bounded by the curve $y = \sec x + \tan x$, the x-axis, and the lines $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$. Find the volume generated when R is rotated about the x-axis completely. [6]

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \pi \left(\sec x + \tan x \right)^2 dx$$

$$= \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\sec^2 x + 2 \sec x \tan x + \sec^2 x - 1 \right) dx$$

$$= \pi \left[2 \tan x - x + 2 \sec x \right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= \pi \left(\left[2 - \frac{\pi}{4} + 2\sqrt{2} \right] - \left[-2 + \frac{\pi}{4} + 2\sqrt{2} \right] \right)$$

$$= \pi \left(4 - \frac{\pi}{2} \right)$$

7.632

$$\frac{1}{x^2 + a^2} = \frac{1}{a^2 \left[1 + \left(\frac{x}{a} \right)^2 \right]} = \frac{1}{a^2} \cdot a \cdot \tan^{-1}\left(\frac{x}{a} \right)$$

3. A curve is such that $\frac{dy}{dx} = e^{2x} - 2e^{-x}$. The point $(0, 1)$ lies on the curve.

(i) Find the equation of the curve. [4]

$y = \int \frac{dy}{dx} dx = \frac{1}{2} e^{2x} + 2e^{-x} + C$

$1 = \frac{1}{2} + 2 + C \Rightarrow C = -\frac{3}{2}$

$y = \frac{1}{2} e^{2x} + 2e^{-x} - \frac{3}{2}$

(ii) The curve has one stationary point. Find the x-coordinate of this point and determine whether it is a maximum or a minimum point. [6]

$\frac{dy}{dx} = 0 \Rightarrow e^{2x} - 2e^{-x} = 0$

$e^{-x}(e^{3x} - 2) = 0$

$\Rightarrow e^{3x} = 2 \Rightarrow x = \frac{1}{3} \ln 2$

$\frac{d^2y}{dx^2} = 2e^{2x} + 2e^{-x}$

$x = \frac{1}{3} \ln 2, \quad \frac{d^2y}{dx^2} > 0,$

So it is a minimum.

THE END